7th International Conference on Reliability and Safety Engineering (ICRSE2023)

Remaining Useful Life Prediction for a Multi-Component System with Degradation Interactions

Mahdiyeh Kalaei [1*], Mohammad Ali Saniee Monfared [2]

[1] **PhD Student at Industrial Engineering Department, Alzahra University, Tehran, Iran; m.kalaei@alzahra.ac.ir**
[2] **Associate Prof., Department of Industrial Engineering, Alzahra University, Tehran, Iran; mas_monfared@alzahra.ac.ir**

Abstract
Remaining useful life (RUL) prediction is an important part of prognostics and health management (PHM). The focus is to predict the time to failure (TTF) or to anticipate the RUL of a system.

This paper is concerned with the prognostics of a multi-component system, in which the degradation processes of the system are affected by both the internal factors of the components and the environment as the external factors.

Keyword: multi-component system, Prognostic, degradation modeling, remaining useful life prediction.

Introduction

In the real industrial process, systems are usually constituted by some components, which may influence each other. Neglecting these dependencies between components will lead to inefficient RUL prediction. In this paper, we consider a multi-component system and present an algorithm for RUL prediction. This paper is organized as follows. In section 2 we review the literature for multi-component systems and consists of three parts: Past review paper, reviewing Multi component models, and Mathematical models available in reviewed articles. In section 3 a multi-component system is considered and modelled, and the model is solved in section 4. The paper is concluded in Section 5.

Multi-component systems review

Past review paper

In the field of maintenance in multi-component systems, three review articles have been written so far. The first one is a review of multi-component maintenance models with economic dependency by Dekker et al (1997). Models are divided as stationary and dynamic models. In stationary models, a long-term stable situation is assumed for an infinite planning horizon and divided into three sub categories of corrective maintenance, preventive/plannable maintenance, and opportunistic maintenance. On the other hand, the dynamic models deal with short term horizon and are divided into two sub categories of finite horizon and rolling horizon. Papers reviewed by Dekker et al fall into these four categories. In other word, they considered four classes of problems and models when an investigator is going to deal with multi-component systems.

The second review paper is by Nicolai and Dekker (2008) who kept the sub categories proposed in the first paper but developed a new understanding of the dependence/interaction that may exist between components in a system. They have three different types: economic, structural, and stochastic. Thus, papers are reviewed accordingly.

The third review paper deals with how to select a maintenance plan for multi-component systems, i.e., by Cao et al (2018). We may consider this paper as the first systematic review paper on multi-component systems maintenance which takes system characteristics, maintenance characteristics, and mission profile characteristics into considerations.

Reviewing multi component models

Predicting the RUL for multi-component systems requires an accurate estimation of the degradation states of constituent components, considering economic dependency, structural dependency, and stochastic dependency.

Economic dependence means that performing maintenance on a certain group of components can save costs and/or time instead of doing it separately. Structural dependence means that the maintenance of a failed component needs the maintenance of other components in the system. Stochastic dependence occurs if the failure of a component influences the states of other components. The existing research papers can be divided into three categories based on the number of dependent factors considered:

One factor systems

Zhang et al (2016) consider a stochastic dependence in the modeling of a multi-component system. Here two Brownian motions are adopted in the degradation process of every component to describe the public noise and the private noise separately. The degradation states and model unknown parameters are first identified recursively by the Kalman filter and expectation-maximization (EM) algorithm. Then the RUL distribution of every component can be predicted by inferring the first hitting time (FHT) with a known threshold. This paper has considered a multi-component system with public noise only using a Brownian motion. An EM algorithm is adopted to identify the parameters of the model, realizing the fact that the EM algorithm can cope with incomplete or missing data. Then Kalman filter was implemented along with EM algorithm to estimate the hidden degradation process of the public noise. Bian et al (2014) considered a stochastic dependence in the modeling of a multi-component system. This is achieved by modeling the behaviors of condition/degradation-based sensor signals that are installed on each component. In addition, a Bayesian framework is used to update the predicted RULs using sensor signals.

Shey et al (2015) modelled stochastic dependency in a multi-component system that consists of two major units, A and B, each subject to two types of shocks occurring according to a non-homogeneous Poisson process. A type II shock causes a complete system failure which is corrected by a replacement and a type I shock causes a unit A minor failure, which is rectified by a minimal repair. The shock type probability is age-dependent. Each unit A minor failure results in a random amount of damage to unit B. Such damage to unit B can be accumulated to a specified level of complete system failure.

Zhang et al (2018) considered stochastic dependency by considering a two-component system with failure interactions. Component 1 is repairable and component 2 is nonrepairable and is subject to increasing degradation. They considered two different types of shock models. In model 1, component 1 failure causes random gradual damage to component 2 and increases its degradation level. In model 2, component 1 failure may cause the failure of component 2 with a given probability while the failure of component 2 is catastrophic and induces the failure of the whole system. For each model, three maintenance policies are considered and evaluated. In each policy, component 1 undergoes imperfect corrective maintenance actions and component 2 is perfectly repaired.

Dinh et al (2020) consider a structural dependency and investigated the impact of disassembly operations on the degradation processes and reliability of the components/system. The disassembly operations and their impact on the components' degradation processes are first analyzed. Next, a model based on a connection matrix is proposed to quantify the impact of disassembly operations on the degradation processes of the components. The proposed model allows considering different influencing factors such as the property of the component, the strength of the connection between the components as well as the degree of expertise of the technician and tool used to perform the disassembly operations. Then, the impact of disassembly operations on the system reliability is analyzed and formulated for a multi-component. Finally, a numerical example of a gearbox system is introduced to illustrate the use and the advantages of the proposed models in the reliability assessment and maintenance optimization framework.

Bakir et al (2021) considered a stochastic dependence in the modeling of a multi-component system. In this paper, an integrated framework that combines i) real-time degradation models used for predicting the remaining life distribution of each component, with ii) mixed integer optimization models and solution algorithms used for identifying optimal wind farm maintenance and operations is proposed. In doing so, a novel optimization model, MC-CBOM, is developed, that adapts to real-time sensor information while accounting for complex dependencies across components, turbines, and wind farms. To ensure scalable deployment of the proposed model, a solution algorithm that exploits the structure of the reformulated optimization model is devised to obtain the optimal O&M policy efficiently. A simulation framework based on the rolling horizon methodology is used to extensively test the performance of the proposed optimization model in terms of net profit, maintenance costs, number of failures, unused life, and turbine availability. It is quantitatively demonstrated that the proposed framework provides significant cost and reliability improvements over existing maintenance models. Furthermore, the ability of the proposed model to adapt to a wide range of operational and maintenance scenarios is illustrated. More specifically, it is observed that i) the MC-CBOM model adapts to increasing electricity prices by pursuing a more aggressive maintenance strategy to ensure high turbine availability, and ii) the multi-component nature of MC-CBOM is effective in prioritizing critical components over others to reduce overall maintenance expenditure. Maintenance decisions identify optimal times to repair every component, which in turn, determines the failure risk of the turbines. More specifically, optimization models that characterize a turbine's failure time as the first time that one of its constituent components fails- a systems reliability concept called competing risk is developed. The resulting turbine failures impact the optimization of wind farm operations and revenue. Extensive experiments conducted for multiple wind farms with 300 wind turbines - 1200 components - showcase the performance of the proposed framework over conventional methods.

Özgür-Ünlüakın et al (2021) considered a stochastic dependence in a complex system with stochastic dependent components which cannot not observed directly, making prediction difficult. A dynamic Bayesian network-based maintenance decision framework was developed to evaluate proactive maintenance policies for such systems. Two preventive and one predictive maintenance strategies from a cost perspective are considered. The performances of the policies are compared with a reactive maintenance strategy and also with each other using different strategy parameters on a real-life system confronted in thermal power plants for six different scenarios. The scenarios are designed considering different structures of system dependability and reactive cost. The results show that threshold-based maintenance which is the predictive strategy gives the minimum cost and maintenance number in almost all scenarios.

Niu et al (2022) investigated stochastic dependency in the modeling of a multi-component system. The objective was to investigate the effects of the stochastic dependence between components on the degradation process and remaining useful life (RUL) of a system. Firstly, a degradation model integrating the effects of stochastic dependence between components was formulated. Then, the probability density function of the RUL was derived for multi-component systems with different structures. Finally, the dependent degradation state and unknown parameters of the model were estimated simultaneously and recursively using Kalman filtering in conjunction with the expectation-maximization algorithm. The superiority of the presented method was confirmed by considering a numerical example and performing case studies of an aircraft turbine engine and a gearbox system.

Moreover, Zhang et al (2022) also considered a stochastic dependence on the modeling of a multi-component system. In this paper, a condition-based maintenance of a two-component system under imperfect inspection is studied: component 1 is repairable and only two states (working and failed) are observable. Component 2 degrades with time. The degradation of component 2 is modeled by a Wiener process with different drift and volatility parameters depending on the working state of component 1. Component 2 fails if its degradation level exceeds a predefined threshold. A dual periodical inspection policy is proposed to monitor the system condition: component 1 is imperfectly inspected and the whole system is perfectly inspected for a longer period. Upon inspection, component 1 is replaced if a failure is detected. The whole system is renewed/replaced if the degradation of component 2 exceeds its preventive maintenance threshold. The problem is very complex due to the failure dependence between the two components and the partially revealed information within the system inspection period. Component 1 is impacted by imperfect inspection. Component 2 may experience multiple changes in its degradation regime while only imperfect, partial change information is available. By taking the maintenance cost rate in the long run as the objective function, the Markov renewal process is implemented to solve the problem. Numerical examples show the applicability of the model.

Meanwhile, Nguyen et al (2022) are among the people who chose a Structural dependency .They presented a probabilistic deep learning methodology for uncertainty quantification of multi-component systems' RUL. It is a combination of a probabilistic model and a deep recurrent neural network to predict the components' RUL distributions. Then, using the information about the system's architecture, the formulas to quantify system reliability or system-level-RUL uncertainty are derived. The performance of the proposed methodology is investigated through the benchmark data provided by NASA.

Li et al (2022) also considered a stochastic dependence. In this paper, a multi-component system with hierarchical stochastic dependencies is considered. A system with four components and hierarchical dependencies is studied. Components within subsystems are strongly correlated and dependence between subsystems is heterogeneous. It is shown that the Nested Clayton Lévy copula enables the modeling of a time-independent hierarchical stochastic dependence structure. Based on the multiple sources of dependence among components and hierarchical structures, a novel condition-based maintenance policy is proposed. The inspection scheme as well as maintenance decision is dynamically planned according to the collected inspection information and the influence of stochastic dependence is investigated. Two condition-based maintenance policies ($policy$ 1 and $policy$ 2) with different emphasis on the availability of information or saving inspection cost are proposed for a four-component system with hierarchical dependencies. To investigate the influence of stochastic dependence, numerical experiments for three cases of systems with different dependence configurations are done and the performance is analyzed and compared to two condition-based maintenance strategies that do not consider stochastic dependence ($policy$ 3 and $policy$ 4). The results show that the proposed condition-based maintenance policy applying the new inspection/replacement strategy is cost-efficient regarding the global inspection cost as well as preventing failures by scheduling inspections according to the individual condition of the component. Even though this work focuses on profiting from stochastic dependence, the proposed maintenance policies group the inspections and replacements at certain times with non-periodic schemes so that more grouping opportunities are provided.

Two factor systems

Shi et al (2016) considered stochastic and economic dependencies together in the modeling of a multi-component system. They presented a dynamic opportunistic condition-based maintenance strategy for multi-component systems. The strategy was based on real-time predictions of the remaining useful life under the simultaneous consideration of economic and stochastic dependence. The remaining useful life of components that have a stochastic dependence on one another is predicted using stochastic filtering theory. Given the condition monitoring history data, they modelled the effect of a component's degradation level on the remaining useful life of other components. A penalty cost was also considered to represent the additional cost of shifting the maintenance time. An optimization model was then established by choosing the dynamic opportunistic maintenance zone and optimal group structure that minimizes the long-term average maintenance cost of the system. A numerical example including three multi-component systems was presented. In developing a preventive maintenance policy for multi-component systems, the following assumptions were taken: components are divided into three categories (A, B, and C) based on their interdependence characteristics: (a) component k in category A: it is assumed that the RUL of component k is influenced by its inherent degradation as well as the degradation of other components. (b) component j in category B: the stochastic dependence of the components is unidirectional. That is the performance degradation of component j influences the RUL of the other components. However, the RUL of component j is not affected by the degradation of other components. (c) component m in category C: the degradation of component m is independent of the other components. The authors developed an approximate methodology for RUL prediction using a stochastic filter. The effect of a component's degradation level on the remaining useful life of other components is considered using real-time CM information. They also presented a dynamic OM model for multi-component systems based on the prediction of RUL under economic and stochastic dependence minimizing the long-term average cost.

Nguyen et al (2017) also considered stochastic and economic dependence. Maintenance and inventory optimizations are two interrelated processes but are often investigated separately in the literature. Joint optimization of these processes can reduce the total maintenance and inventory cost, it may, however, lead to a complex problem with a large number of decision parameters to be optimized. To face this open issue, they presented a joint predictive maintenance and inventory strategy for systems with complex structures and multiple non-identical components. Both predictive maintenance and spare parts provisioning operations are studied and optimized jointly. The prognostic condition index and the structural importance measure of components are jointly used to build the thresholds for preventive maintenance and spare part orders. Additionally, to take the advantage of economic dependence, opportunistic maintenance decision rules based on the criticality level of components and their spare parts availability are proposed. To evaluate the proposed joint strategy, a cost model taking into account the economic dependence of both maintenance and inventory activities is developed.

Shen et al (2018) considered stochastic and structural dependence in the modeling of a multi-component system. They studied the reliability of a multi-component system with interacting components that are subject to both continuous degradation processes and categorized shocks. By interacting components, we refer to the case where the degradation behavior of a particular component can influence that of another component. This is in contrast to previous works in the literature that typically assume independence in the degradation interaction in multi-component systems. Moreover, categorized shocks are assumed to selectively affect one or more components by either causing a sudden jump in the degradation level or accelerating the degradation rate, or both. Under these assumptions, they derived the reliability of a series system recursively and propose a simulation method to approximate the failure time of k -out-of- n systems. An example of the MEMS (Micro-electromechanical systems) oscillator, which is a typical system with interacting components subject to continuous degradation and categorized random shocks, is studied in the end to demonstrate the results obtained in this paper. Different types (or categories) of shocks selectively affect one or more components in two ways: either by causing a sudden jump in degradation level or by accelerating the degradation rate, or both.

By interacting components, they meant that the degradation behavior of a particular component can influence that of another component. We modeled this aspect by introducing an interaction matrix C that captures the interaction relationships between different components and can be dynamically updated when the components' interactions change during the system's operation. The shared exposure of components to the same shocks and the interaction relationships presented a set of interesting and challenging aspects in analyzing and deriving system reliability. To address these challenges, they introduced a Markov renewal process to depict the arrival process of random shocks, which is a more suitable choice than Poisson processes when shocks are categorized into different types.

Martinod et al (2018) considered stochastic and economic dependence. This article proposes a stochastic optimization model to reduce the long-term total maintenance cost of complex systems. The proposed work is based on the following approaches: i) optimization of a cost model for complex multi-component systems consisting of preventive and corrective maintenance using reliability analysis, which faces two different maintenance policies (periodic block-type and age-based), and ii) a clustering method for maintenance actions to decrease the total maintenance cost of the complex system. This work evaluates each maintenance policy and measures the effects of imperfect maintenance actions. They considered a model for both different preventive maintenance policies (periodic block-type and age-based). For corrective maintenance, the repairmen's action should be taken immediately after the failure of the component, the proposed methodology adopts an ABAO corrective maintenance policy.

Xu et al (2018) considered the aspect of stochastic and economic dependencies in the modeling of a multi-component system. In this research, state-rate dependence denoting interaction between component health condition (degradation state) and the failure rate is proposed for degradation and failure analysis for a two-component repairable system. A state discretization technique is proposed to model how the health state of one component affects the hazard rate of another. An extended proportional hazard model (PHM) is used to characterize the failure dependence and estimate the influence of the degradation state of one component on the hazard rate of another. An optimization model is developed to determine the optimal hazard-based threshold for a two-component repairable system. Our proposed technique integrates both degradation and event (failure) data jointly and the assumptions for our work are given below:

• All the components are subject to monotonic non-decreasing degradation,

the component hazard rate is a function of time and the observed internal and external covariates;

• Failure can happen at any instant; component failure makes the system stop;

• Minimal repair can restore the system's full functionality and does not affect the state of the repaired system. Replacement is perfect and restores the system to be as good as new, repair and replacement time are negligible;

• Condition monitoring is continuous and failure can only be detected at the inspection. The inspection cost is negligible.

Do et al (2018) considered stochastic and economic dependence. They developed a model of a condition-based maintenance policy for a two-component system with both stochastic and economic dependencies. The stochastic dependency is such that the degradation rate of each component depends not only on its state (degradation level) but also on the state of the other component. The economic dependency is such that combining multiple maintenance activities has a lower cost than performing maintenance on components separately. To select a component or components to be preventively maintained, adaptive preventive maintenance and opportunistic maintenance rules are proposed. A cost model is developed to find the optimal values of decision variables. A case study of a gearbox system demonstrates the utility of the proposed model.

A cost model taking into account the economic dependence between components is developed to find the optimal values of the decision variables. The policies are studied in the context of a gearbox system consisting of gears.

The results indicate that i) accounting for the state dependence between components is important, and ignoring it has a significant impact (29.3%) on the cost; ii) introducing an opportunistic threshold for replacement makes the maintenance policy more flexible and less sensitive to a sub-optimally large inspection interval; and iii) when there exists positive stochastic dependence between components so that components tend to deteriorate together, introducing an opportunistic threshold for a replacement to share set-up costs achieves less when there is positive stochastic dependence between components than when there is not. This is because replacements will tend to be synchronized and this tendency to synchronize arises precisely because of degradation dependence.

Shafiee et al (2019) also considered stochastic and economic dependence. This study presents a combined analytic network process and cost-risk criticality analysis model to select a cost-effective, low-risk maintenance strategy for different sets of components of a complex system. The proposed model consists of four maintenance alternatives (i.e., failure-based, time-based, risk-based, and condition-based), among which the most appropriate strategy, based on two criteria of maintenance implementation costs and failure criticality, is to be chosen. the former criterion includes the annual maintenance expenditure required for hardware, software, and personnel training, while the latter criterion focuses on the capability of maintenance in reducing the failure vulnerability and enhancing reliability and resilience. The possible dependencies among selection criteria as well as the failure interactions between components are taken into account to evaluate the maintenance alternatives. Finally, the model is applied to determine a suitable maintenance strategy for a new wind turbine configuration consisting of several mechanical, electrical, and auxiliary components at the design stage. The results are compared with practices of maintenance over the first year of system operation as well as with the results obtained from an analytic hierarchy process model. A combined ANP and 'cost-risk criticality analysis' model is proposed aiming to select a cost-effective, low-risk maintenance strategy for different sets of components of a complex system. The possible dependencies among selection criteria as well as the interactions between component failures (cascade effects) are taken into account to evaluate the maintenance alternatives. The proposed model consists of two sets of criteria, namely, cost of maintenance and criticality of failure. The former criterion includes the annual maintenance expenditure required for hardware, software, and personnel training, while the latter focuses on the capability of maintenance in mitigating the failure vulnerability and increasing reliability and resilience. The model is applied to determine a suitable maintenance strategy for a new wind turbine configuration consisting of several mechanical, electrical, and auxiliary components at the design stage. To the best of the authors' knowledge, this article is the first academic paper dealing with the maintenance strategy selection of wind power systems using the MCDM techniques. Our analysis results revealed that Among the strategies of CBM, TBM, RBM, and FBM, the condition-based and RBM strategies were the two preferred strategies for wind turbine power systems.

Zhang et al (2020) also considered stochastic and economic dependencies. Most existing research on CBM assumes that preventive maintenance should be conducted when the degradations of system components reach specific threshold levels upon inspection. However, the search for optimal maintenance threshold levels is often efficient for low-dimensional CBM but becomes challenging if the number of components gets large, especially when those components are subject to complex dependencies. To overcome the challenge, they proposed a flexible CBM model based on customized deep reinforcement learning for multi-component systems with dependent competing risks. Both stochastic and economic dependencies among the components are considered. Specifically, different from the threshold-based decision-making paradigm used in traditional CBM, the proposed model directly maps the multi-component degradation measurements at each inspection epoch to the maintenance decision space with a cost minimization objective, and the leverage of deep reinforcement learning enables high computational efficiencies and thus makes the proposed model suitable for both low and high dimensional CBM. In this article, we consider two widely used stochastic processes that satisfy desired assumptions to model the SDPs, i.e., the Compound Poisson Process (CPP) and Gamma Process (GP). This article presents the detailed simulation process of agent-environment interactions in the DRL approach.

Liu et al (2021) attention to the aspect of stochastic and economic dependence in the modeling of a multi-component system. This paper analyzes a condition-based maintenance (CBM) model for a system with two heterogeneous components in which degradation follows a bivariate gamma process. In the proposed CBM policy, both components are periodically inspected and a preventive or corrective replacement might be carried out based on the state of degradation at inspection. The components are subject to periodic inspection and will be preventively replaced if their degradation levels exceed PM thresholds. The CBM model is formulated as a Markov decision process (MDP) and dynamic programming is used to compute the expected maintenance cost over a finite planning horizon. The expected maintenance cost is minimized concerning the preventive replacement thresholds for the two components. Unlike an infinite-horizon CBM problem, which leads to a stationary maintenance policy, the optimal policy in the finite-horizon case turns out to be non-stationary in the sense that the optimal actions vary at each inspection epoch. For a relatively long but still finite planning horizon, a hybrid policy that combines the stationary and dynamic policies is suggested, i.e., engineers can follow the stationary policy at the beginning and switch to the dynamic policy when approaching the end of the horizon. a numerical example is presented to illustrate the proposed model and investigate the influence of stochastic and economic dependency and the correlation between the degradation processes on the optimal maintenance policy. Numerical results show that a higher dependence between the degradation processes reduces the maintenance cost, while a higher economic dependence leads to higher preventive replacement thresholds.

Xu et al (2021) also considered stochastic and economic dependence. This paper aims to investigate the optimal CBM policy under periodic inspection for a K-out-of-N: G system, where economic dependency, stochastic dependency, and imperfect maintenance are emphasized. The objective is to minimize the expected long-run discounted cost. In the model, the cumulative degradation of each component is modeled by heterogeneous stochastic processes, the dependence among all components is characterized by a copula function, and the imperfect maintenance is represented by a reduction in the degradation level. Since the system has Markov property, we solve the CBM optimization problem based on the Markov decision process (MDP) framework. To ease the computation burden, we discretize the continuous state space and then use the value iteration algorithm with Monte Carlo simulation to find the optimal inspection interval and the optimal CBM policy. Numerical studies for a 1-out-of-2: G system are conducted to systematically examine the impacts of degradation processes, copula functions, and imperfect maintenance on optimal maintenance decisions, which provides insights for multi-component system maintenance. A sensitivity analysis of cost-related parameters is also performed.

Kıvanç et al (2022) are among the people who chose stochastic and structural dependencies. Partially Observable Markov Decision Processes (POMDPs) are powerful tools for such problems under uncertainty in stochastic environments. In this study, the main POMDP solution approaches and solvers are surveyed. Then, based on experimental models with different complexities in the size of the system space, selected POMDP solvers using different representation patterns for modeling and different procedures for updating the value function while solving are compared. we apply factored POMDP to a case study on the one-line RAH system which has stochastic and structural dependencies among its components. Furthermore, to show that factored representations are advantageous in modeling and solving the maintenance problem of multi-component systems where there exist also stochastic dependencies among the components, the maintenance problem of the one-line regenerative air heater system available in thermal power plants is modeled and solved with factored POMDPs. In-depth sensitivity analyses are performed on the obtained policy. The results show that factored POMDPs enable compact modeling, efficient policy generation, and practical policy analysis for the tackled problem.

Tambe (2022) also considers stochastic and economic dependence. In this paper, a selective maintenance decision model to determine maintenance actions namely, repair, replace or do nothing for a multicomponent system is presented. The objective function is to minimize the total cost of maintenance decisions. A simulated annealing algorithm is used for the optimization of the decision variables. Finally, a numerical example is presented to show the applicability of the model.

Three factor systems

Wang et al (2022) considered stochastic, economic, and structural dependencies and presented a bi-level approach to optimize a CBM policy for multi-component systems subject to stochastic and economic dependencies. At the component level, the aim is to identify the optimal group of components to be preventively maintained when maintenance is triggered due to the decision made at the system level. The optimal inter-inspection interval, the thresholds of the system, and component conditional reliability for triggering maintenance are jointly determined for minimization of the long-run average cost rate. They developed a mathematical maintenance cost optimization model based on predictive reliability data of both system and components and derive the optimal maintenance policy through the Monte Carlo simulation technique and a newly designed PSO-based heuristic algorithm. The multi-component system considered in the paper has a k-out of- n: G structure.

The system was built from n non-identical deteriorating components. The degradation path of each component evolves stochastically depending on the way that the change in the degradation state of each component affects the degradation rates of other components. To simplify the problem, we consider only the case of perfect observations without including measurement error. Despite the stochastic dependence through the state–state interactions and state–rate interactions, herein we investigate how the gradual degradation increment of one component affects the degradation rate of other components continuously over time. As a result, the stochastic dependence is modeled through degradation rate–rate interactions .In the present study, we assume that the values of δhj are fixed, and so are the ones of μj and σj.

Oakley et al (2022) also considered stochastic, economic, and structural dependencies. In this paper, we propose a condition-based maintenance policy for continuously monitored multi-component systems subject to stochastic dependence through load sharing and economic dependence through maintenance set-up costs. In this paper, we propose a novel, condition-based maintenance policy to obtain optimal replacement decisions at maintenance opportunities. Through numerical studies, they showed the importance of a policy that incorporates both types of system dependence. The policy incorporates a utility/reward function that is a combination of interpretable penalties that encapsulate the costs of stochastic and economic dependence. The utility function trades off the rewards of clustering components with the loss due to load sharing. The policy minimizes the overall cost of the system by choosing actions that minimize the total long-term penalty. We show that the proposed policy outperforms various alternative policies by reducing system life-cycle costs. The proposed policy suggests a coherent way to incorporate uncertainty and proposes a way to model component failure times using a random degradation threshold. Through simulation, maintenance clustering is especially beneficial for systems with a strong degree of economic dependence. Furthermore, they observed that replacements can be performed immediately upon component failure for systems with a high degree of stochastic dependence (compared to the degree of economic dependence), as more cost savings can be obtained from stochastic dependence than through economic dependence.

Numerical examples showed that the CBM policy outperforms the individual maintenance policy, which prevents loss due to stochastic dependence, and the simultaneous maintenance policy, which prevents loss through economic dependence, by reducing system life-cycle costs.

Zhou et al (2016) are among the researchers who chose stochastic, economic, and structural dependencies for modeling the multi-component system. Maintenance optimization of a parallel-series system considering both stochastic and economic dependence among components as well as limited maintenance capacity is studied in this paper. The maintenance strategies of the components are jointly optimized, and the degradation process of the system is modeled to address the stochastic dependence and limited maintenance capacity issues. To overcome the “curse of dimensionality” problem where the state space of a parallel-series system increases rapidly with the increased number of components in the system, the factored Markov decision process (FMDP) is employed for maintenance optimization in this work. An improved approximate linear programming (ALP) algorithm is then developed. The selection of the basic functions and the state relevance weights for ALP is also investigated to enhance the performance of the ALP algorithm. Results from the numerical study show that the current approach can handle the decision optimization problem for multi-component systems of moderate size, and the error of maintenance decision-making induced by the improved ALP is negligible. The outcome of this research provides a useful reference to overcoming the “curse of dimensionality” problem during the maintenance optimization of multi-component systems.

The details of the above reviewed papers are summarized in Table 1.

Table 1. Reviewing Multi component models

author	system structure	dependencies	The distribution used in modeling	interaction	aim	solution method	Environmental influence	Description
Zhang et al (2016) [4]	-	stochastic	The RUL of every component can be derived using the FHT (first hitting time).	the degradations are affected by a common factor which is assumed to be public noise	presents a methodology to predict the RUL of a class of multi-component systems with hidden degradation processes	The degradation states and model unknown parameters are first identified recursively by the Kalman filter and EM algorithm. the RUL distribution of every component can be predicted by inferring the first hitting time (FHT)	*	PHM
Bian et al (2014) [5]	-	stochastic	Brownian motion, Gamma process	degradation-rate	predict the residual lifetime of a multi-component system	a Bayesian framework is used to update the predicted RLDs, DRI modeling framework, Base-Case DRI Model, MLE is used to estimate the parameter	-	-
Shey et al	-	stochastic	Non-homogeneous Poisson process	Shock Damage Interaction	cost minimization	Cumulative damage model	-	Replacement policy, shock model
Zhang et al (2018) [7]	-	stochastic	Weibull lifetime distribution, Exponential distribution used to describe the damages	failure	long-run average cost optimization	The failure processes of all components are modeled by either their failure rates or their degradation processes, imperfect maintenance (virtual age method),	-	Corrective maintenance, Preventive maintenance, Maintenance cost derivation (T policy, N policy, (N, T) policy)

Dinh et al (2020) [8]	series	structural	The half-normal distribution is herein used to model the adjustment factor, B(t) is standard Brownian motion in the degradation model	-	This paper aims to investigate the impact of disassembly operations on the degradation processes and reliability of the components/system	Dependence modeling and formulation of disassembly impact, Degradation modeling with disassembly operations impact, Reliability assessment considering the disassembly operations impacts	-	Step 1: System structure analysis (component analysis, dependencies analysis…)- Step 2: Dependence modeling and formulation of disassembly impact (Directed graph, connection matrix, disassembly impact model)- Step 3: Degradation modeling with disassembly impact (degradation models, impacts model)- Step 4: Reliability assessment considering the disassembly operations impacts (maintenance policy, reliability model)
Bakir et al (2021) [9]	-	stochastic	Bayesian approach	-	The proposed framework provides significant cost and reliability improvements. the proposed model adapts to a wide range of operational and maintenance scenarios.	The MC-CBOM policy outperforms all considered benchmark policies thanks to its ability to consider and adapt to the complex interactions between different decision layers.	-	CBM, Large-scale mixed integer optimization
Niu et al (2022) [10]	Series structure, Parallel structure, Summation structure	Stochastic	B(t) indicates standard Brownian motion, observation noise, follows a normal distribution	Degradation,	The objective of this study was to investigate the effects of the stochastic dependence between components on the degradation process and remaining useful life (RUL) of a system.	The PDF of the RUL was derived for a multi-component system using the FHT concept, The dependent degradation state and unknown parameters of the model were jointly estimated by Kalman filtering (KF), and the expectation maximization (EM) algorithm. the maximum likelihood estimation (MLE)	*	state-space model

Özgür-Ünlüakın et al (2021) [11]	The RAH system consists of two parallel motors groups	Stochastic	-	-	To reduce maintenance costs while increasing system reliability at the same time	Dynamic Bayesian networks, Tabu procedure, Generic algorithm for the proactive maintenance strategies	-	Multi-component hidden systems
Zhang et al (2022) [12]	-	stochastic	The degradation of component 2 is described by a Wiener process with parameters depending on the state of component 1, the lifetime of component 1 follows a Weibull distribution	Component 2 fails if the degradation level of component 1 exceeds a predefined threshold, The whole system is renewed if the degradation of component 2 exceeds its preventive maintenance threshold	The expression of the maintenance cost rate, in the long run, is given to evaluate the maintenance policy	An algorithm is proposed to achieve maintenance optimization (The calculation of the maintenance cost in the long-run horizon), By taking the maintenance cost rate in the long run as the objective function, the Markov renewal process is implemented to solve the problem.	-	condition-based maintenance of a two-component system under imperfect inspection,
Nguyen et al (2022) [13]	Series, parallel, combine d, bridge-type	multi-independent-component systems, Structural	lognormal distribution, and a recurrent neural network	-	To develop an efficient approach able to provide the pdf of the RUL and outperform existing methods for both point-wise and probabilistic RUL predictions with a reasonable computational cost	A combination between a probabilistic model, i.e., lognormal distribution, and a recurrent neural network, i.e., LSTM model	-	PHM
Li et al (2022) [14]	Each subsyste m is parallel.	hierarchical stochastic dependencie s	Individual degradation is modeled by the Gamma process, the Inverse Gauss (IG) process with Gaussian the copula is used when modeling the deterioration process of heavy machine, Lévy copula permits the marginal to be Gamma process and the Wiener process	-	reducing the inspection and maintenance cost, the optimal maintenance cost of policies	the Nested Lévy copula, Copula methods in dependent degradation modeling	-	Condition-based maintenance policy, Nested Lévy copula

Shi et al (2016) [15]	-	stochastic and economic	The system's initial RUL is modeled as a Weibull distribution	The components are divided into three categories (A, B, and C) based on their interdependence characteristics	minimization of the long-term average maintenance cost of the system	An approximate methodology for RUL prediction using a stochastic filter	-	dynamic maintenance strategy, OM policy
Zhang et al (2020) [16]	-	stochastic and economic	Poisson Process (CPP) and Gamma Process (GP)	agent-environment	cost minimization objective	A new CBM model for a K-component system subject to dependent competing risks, which are general and different from existing models	*	CBM, a novel and flexible CBM model based on a customized deep reinforcement learning for multi-component systems with dependent competing risks
Shen et al (2018) [17]	series, k-out of- n	Stochastic, Structural	degradation between two adjacent shocks is governed by the Gamma process,	degradation (the degradation behavior of a particular component can influence that of another component), Moreover, categorized shocks are assumed to selectively affect one or more components by either causing a sudden jump in the degradation level or accelerating the degradation rate, or both.	The key contribution of this work is studying the reliability of a multi-component system with interacting components subject to continuous degradation processes and categorized shocks	Markov renewal process	-	a Markov renewal process
Martinod et al (2018) [18]	-	stochastic and economic	The stochastic hazard rate of each component has a uniform distribution	state-rate	to reduce the long-term total maintenance cost of complex systems	Periodic block-type policy, Age-based policy, an ABAO corrective maintenance policy	-	preventive and corrective maintenance policy

Xu et al (2018) [19]	component failure makes the system stop	stochastic and economic	proportional hazard model (PHM) with baseline Weibull hazard function and time-dependent stochastic covariates is used to describe the equipment deterioration process	state-rate	effective degradation analysis and accurate condition assessment, cost minimization	A state discretization technique to model how the health state of one component affects the hazard rate of another, an extended proportional hazard model (PHM) to characterize the failure dependence and estimate the influence of the degradation state of one component on the hazard rate of another, an optimization model is developed to determine the optimal hazard-based threshold for a two-component repairable system	-	PHM (proportional hazard model), CBM
Nguyen et al (2017) [20]	combined	Economic, stochastic	Gamma distribution, Weibull distribution	cost-rate	To reduce the total maintenance and inventory cost	present a joint predictive maintenance and inventory strategy for systems with complex structures and multiple non-identical components, opportunistic maintenance decision rules based on the criticality level of components and their spare parts availability are proposed, an adaptive maintenance opportunity rule, Monte Carlo simulation techniques	-	predictive maintenance, predictive inventory, prognostic, opportunistic maintenance, Joint predictive maintenance, and inventory strategy
Do et al (2018) [21]	-	Economic, stochastic	gamma distribution, Brownian motion process	economic and degradation	cost optimization	A particle filter is implemented to estimate the parameters of the proposed model, State dependence modeling, and the cost rate is evaluated, using Monte Carlo simulation	-	Condition-based maintenance, an opportunistic maintenance policy, cost of preventive and corrective replacement, preventive and opportunistic maintenance
Shafiee et al (2019) [22]	combined	stochastic and economic		interactions between component failures	model is proposed to select a cost-effective, low-risk maintenance strategy for different sets of components in a complex system	AHP, ANP, fuzzy logic	-	MCDM: CBM-TBM- RBM-FBM, the proposed model consists of two sets of criteria, namely, cost of maintenance and criticality of failure

Liu et al (2021) [23]		stochastic and economic	degradation follows a bivariate gamma process	Degradation (correlation between the degradation processes)	The expected maintenance cost is minimized concerning the preventive replacement thresholds for the two components	Markov decision process (MDP), dynamic programming is used to compute the expected maintenance cost over a finite planning horizon	-	preventive or corrective replacement, CBM
Xu et al (2021) [24]	K-out-of-N: G system	Stochastic, economic	Wiener process, Gamma process, and IG process	Degradation (Copula function is used to characterize the degradation dependence)	The objective is to minimize the expected long-run discounted cost	Copula, we solve the CBM optimization problem based on the Markov decision process (MDP) framework	-	imperfect maintenance
Kıvanç et al (2022) [25]	The one-line RAH system	stochastic and structural	-	-	A sensitivity analysis is conducted concerning several unit downtime costs, maintenance duration of the components, and probabilities of the exogenous variables	Factored Partially Observable Markov Decision Processes (factored POMDPs)	-	factored POMDPs
Tambe (2022) [26]	-	stochastic and economic	The time-to-failure distribution of each component is assumed to follow a two-parameter Weibull distribution	-	The objective function is to minimize the total cost of maintenance decision	Simulated Annealing Algorithm	-	Selective maintenance: Repair , Replacement , Do-nothing
Wang et al (2022) [27]	k-out of-n	stochastic and economic, Structural	Wiener Processes, Brownian motion, inverse Gaussian	rate–rate	minimization of the long-run average cost rate	The Monte Carlo simulation technique combined with an improved Particle Swarm Optimization (PSO)-based heuristic algorithm	-	CBM policy, Perfect observations without including measurement error
Oakley et al (2022) [28]	the series-parallel system, combined	stochastic and economic, Structural	The workload at every time unit follows the truncated normal distribution, Gamma processes will be used to model the degradation of the components	degradation-rate	minimizes the overall cost of the system	To model component failure times using a random degradation threshold, the maintenance clustering is especially beneficial for systems with a strong degree of economic dependence.	-	CBN policy

Zhou et al (2016) [29]	parallel-series	stochastic and economic and structure	The duration of both corrective and preventive maintenance follows the geometric distribution	-	To overcome the "curse of dimensionality" problem, the cost minimization	the factored Markov decision process (FMDP), An improved approximate linear programming (ALP) algorithm		The cost of preventive and corrective maintenance, Maintenance capacity

Mathematical models

The reviewed articles, in the previous section, have the mathematical models collected in Table 2.

Table 2. Mathematical Model of reviewing Multi component models

author	**Mathematical Model**
Zhang et al (2016) [4]	Every degradation can be described by a stochastic process $$X^{(i)}(t)=\eta_0^{(i)}+\eta^{(i)}t+\xi^{(i)}B(t)+\sigma^{(i)}B^{(i)}(t), i=1,2,\ldots,n \text{ .}$$
Bian et al (2014) [5]	The degradation signal of component i $$S_i(t)=S_i(0)+\int_0^t r_i[v;\kappa_i,\boldsymbol{h}(\boldsymbol{S}(v))]\mathrm{d}v+\epsilon_i(t),$$ $$\boldsymbol{r}(t)=\begin{cases}\boldsymbol{\mu}_1 & t_0\le t<p_1\\ \boldsymbol{\mu}_2 & p_1\le t<p_2\\ \cdots & \cdots\\ \boldsymbol{\mu}_L & p_{L-1}\le t<p_L\\ \boldsymbol{\mu}_{L+1} & p_L\le t\le t_q.\end{cases} \qquad \hat{h}_i(s_i(t))=\begin{cases}0 & s_i(t)<\hat{g}_{i,1}\\ 1 & \hat{g}_{i,1}\le s_i(t)<\hat{g}_{i,2}\\ \cdots & \cdots\\ \widehat{M}_i & s_i(t)\ge\hat{g}_{i,\widehat{M}_i-1}\end{cases}$$
Shey et al (2015) [6]	The probability that type I shocks occur exactly k times during [0, t] $$P_k(t)=P\{M(t)=k\}=\frac{\left(\int_0^t q(x)r(x)\mathrm{d}x\right)^k\exp\left\{-\int_0^t q(x)r(x)\mathrm{d}x\right\}}{k!}$$ A corrective replacement is carried out at the first type II shock. The probability of this event is given by $$P_{II}=\sum_{j=0}^{N-1}P(Z_j<K)\int_0^T P\{M(t)=j\}\,\mathrm{d}F_p(t)=\sum_{j=0}^{N-1}G^{(j)}(K)\int_0^T\left[H^{(j)}(t)-H^{(j+1)}(t)\right]\mathrm{d}F_p(t)$$

Zhang et al (2018) [7]	The average long-run cost $C_{\infty I}(T) = \frac{c_3 - (c_3 - c_2)\sum_{n=0}^{\infty} r^n p_n(T)\bar{G}_{\sigma_L}(T) + \sum_{n=1}^{\infty}(n-1)c_1 r^{n-1}\bar{r}\int_0^T \bar{G}_{\sigma_L}(t)\mathrm{d}V_n(at)}{\int_0^T \bar{F}_{sI}(t)\mathrm{d}t} + \frac{\sum_{n=0}^{\infty} n r^n c_1\left(\int_0^T p_n(t)\mathrm{d}G_{\sigma_L}(t) + p_n(T)\bar{G}_{\sigma_L}(T)\right)}{\int_0^T \bar{F}_{sI}(t)\mathrm{d}t},$
Dinh et al (2020) [8]	The degradation process of component i $X_{Hi}(t) = X_i(t) + \sum_{k=0}^{N(t)} H_{iG^k}$ $H_{iG^k} = \sum_{j=1}^{n} \delta_{ij} \cdot I_{ij}^{G^k}$
Bakir et al (2021) [9]	Degradation model for component k in turbine i $D_{i,k}(t) = \varphi_{i,k}(t;\kappa,\theta_{i,k}) + \varepsilon_{i,k}(t;\sigma)$ The underlying base degradation function for component k of turbine i $\varphi_{i,k}(t;\kappa,\theta_{i,k})$
Niu et al (2022) [10]	The intrinsic degradation process of component i $X^{(ii)}(t) = X^{(ii)}(0) + \int_0^t \mu_i(\gamma,\theta_i)d\gamma + \sigma_B^{(i)}B(t),$ The drift term $\mu_i(t,\theta_i) = \lambda_i r t^{r-1}$
Özgür-Ünlüakın et al (2021) [11]	The joint probabilities of the variables in a Dynamic Bayesian Network $P(X_{1:T}) = \prod_{t=1}^{T}\prod_{i=1}^{N} P(X_t^i \mid Pa(X_t^i)).$
Zhang et al (2022) [12]	The degradation process of component 2 given that component 1 failure occurs once at θ $Y_{\theta,t} \sim \mathcal{N}\left(\mu_1\theta + \mu_2(t-\theta), \sigma_1^2\theta + \sigma_2^2(t-\theta)\right)$
Nguyen et al (2022) [13]	The RUL of component *i* $\prod_{i=1}^{n_s}\prod_{t=1}^{n_t} L(\mu_i^t, \sigma_i^t \mid RUL_i^{*(0:t)}) = \prod_{i=1}^{n_s}\prod_{t=1}^{n_t}\prod_{j=0}^{t} \frac{1}{RUL_i^{*j} \cdot \sigma_i^t \sqrt{2\pi}} \times \exp\left(-\frac{(\ln RUL_i^{*j} - \mu_i^t)^2}{2(\sigma_i^t)^2}\right)$ $RUL_i^{*(0:t)} = [RUL_i^{*0}, RUL_i^{*1}, \ldots, RUL_i^{*t}]$

Li et al (2022) [14]	The deterioration stochastic process of component j in subsystem i can be modeled as follows $X_t^{ij} = \sum_{n=1}^{\infty} U_{ij}^{-1}(\Gamma_n^{ij}/T)\mathbb{1}_{[0,t]}(v_n),$ $(\Gamma_n^{ij})_{n\in\mathbb{N}}$are coupled by a copula function C in such a way that $U(U_{11}^{-1}(\Gamma_n^{11}), U_{12}^{-1}(\Gamma_n^{12}), \ldots, U_{MN_M}^{-1}(\Gamma_n^{MN_M})) = C(\Gamma_n^{11}, \Gamma_n^{12}, \ldots, \Gamma_n^{MN_M})$
Shi et al (2016) [15]	The CM history of component j in D_k until t_i $S_i^{(k)} = \{z_i^{(k)} \leqslant Z_i^{(k)} \leqslant z_i^{(k)} + \Delta z_i^{(k)}, S_{i-1}^{(k)}\}$ $= \{z_1^{(k)} \leqslant Z_1^{(k)} \leqslant z_1^{(k)} + \Delta z_1^{(k)}, z_2^{(k)} \leqslant Z_2^{(k)} \leqslant z_2^{(k)} + \Delta z_2^{(k)}, \ldots, z_i^{(k)} \leqslant Z_i^{(k)} \leqslant z_i^{(k)} + \Delta z_i^{(k)}\}$ The RUL of component k $f_i^{(k)}\left(t \middle\| s_i^{(k)}, s_i^{(j)}\right) = P\left(t \leqslant T_i^{(k)} \leqslant t + \Delta t \middle\| T_i^{(k)} > 0, S_i^{(k)}, S_i^{(j)}\right) / \Delta t, \quad \Delta t \to 0$ The system's initial RUL is modeled as a Weibull distribution $f_0(t) = \alpha\beta(\alpha t)^{\beta-1}\exp(-(\alpha t)^{\beta})$
Zhang et al (2020) [16]	Based on the CPP (Compound Poisson Process) model, the degradation of component i $d_i(t) = \sum_{t_{i,j} \leq t} m_{i,j} + \sum_{t_{s,j} \leq t} m_{s,j}$ The PDF of the degradation increment from time t_1 to t_2 based on GP (Gamma Process) $f_{(t_2\ \ t_1)}(u) = \begin{cases} \frac{(t_2\ \ t_1)_u\ (t_2\ \ t_1)\ 1_e\ \ u}{(\ (t_2\ \ t_1))} & \text{if } u \geq 0; \\ 0 & \text{if } u < 0 \end{cases}$
Shen et al (2018) [17]	The accumulated degradation of component i just after the jth shock arrives $W_j^{(i)} = W_{j-1}^{(i)} + Z(T_j - T_{j-1}) + D_{im}\mathbf{1}\{i \in U_m\},$
Martinod et al (2018) [18]	The weighted average cost for the jth component in the periodic block-type actions is expressed as $\Gamma p_{j,\eta} = Cp_j T_j\left(1 - \frac{\alpha_{p1} + p\,\alpha_{p2}}{1 + p}\right)$ The weighted average cost for the jth component in the age-based policy actions are the following $\Gamma p_{j,\eta} = \frac{Cp_j}{A_j}\left(1 - \frac{\alpha_{p1} + p\,\alpha_{p2}}{p + 1}\right)$

Xu et al (2018) [19]	As to identify the optimal number of change points V*, the contrast function, which measures the quality of the number of segments is given below: $$U(V) = \sum_{l=1}^{N}\left[\frac{1}{n^{(l)}}\sum_{v=1}^{V} n^{(l)}{}_{v}\ \log\|\hat{\Sigma}_{\tau^{(l)}{}_{v}}\| + V\left[\left(V+1\right)c^{(l)}{}_{1} + Vc^{(l)}{}_{2} + \ldots + c^{(l)}{}_{v+1}\right]\right]$$ The total cost C cycle in association with repair and replacement for each regenerative cycle is: $$C_{cycle} = N_1 C_{m1} + N_2 C_{m2} + C_p = C_m + C_p$$ The average cost per unit of time C_{AC} is expressed as follows $$C_{AC} = \frac{C_{cycle}}{E[T_r]} = \frac{N_1(D)C_{m1} + N_2(D)C_{m2} + C_p}{W(D)} = \frac{C_m(D) + C_p}{W(D)}$$
Nguyen et al (2017) [20]	The total cost rate can be rewritten as $$C_{\infty}^{T}(T, \varpi_p, \varpi_o) = \frac{E[\sum_{k=1}^{N} C_k^T]}{E[T_{rep} - D(T_{rep})]},$$ $$C_k^T = \underbrace{C_{T_k}^{insp} + C_{(T_{k-1},T_k)}^{remp} + C_{T_k}^{remp}}_{C_k^M} + \underbrace{C_{(T_{k-1},T_k]}^{order} + C_{(T_{k-1},T_k)}^{hold}}_{C_k^I},$$
Do et al (2018) [21]	The cost rate is defined generally as $$C^{\infty}(\Delta T, m_p^1, m_o^2, m_p^2, m_o^2) = \frac{\mathbb{E}[C^{T_{re}}(\Delta T, m_p^1, m_o^2, m_p^2, m_o^2)]}{\mathbb{E}[T_{re}]},$$ $$C^{T_{re}}(\Delta T, m_p^1, m_o^2, m_p^2, m_o^2) = \frac{\sum_{k=1}^{m}(C_{ins}^k + C_{main}^k) + T_{down}\cdot c_d}{m\cdot\Delta T}$$
Shafiee et al (2019) [22]	The desirability index for alternative i, D_i, is defined as the following equation $$D_i = \sum_{j=1}^{J}\sum_{k=1}^{K_j} C_j M_{kj} A_{ikj}$$ where J is the index set for criterion j, K_j is the index set of sub-criteria for criterion j, C_j represents the relative importance of criterion j, M_{kj} is the relative importance of sub-criterion k of criterion j and A_{ikj} is the rating of alternative i on sub-criterion k of criterion j.
Liu et al (2021) [23]	In a univariate gamma process, degradation at time t, Y (t), follows a Bivariate gamma distribution $$f_{\alpha t,\beta}(y) = \frac{\beta^{\alpha t}}{\Gamma(\alpha t)} e^{-\beta y} y^{\alpha t-1}.$$ $$Y(t) \sim Ga(t;\alpha,\beta)$$

Xu et al (2021) [24]	The degradation process for each component follows a stochastic process with independent increments. The Wiener process, Gamma process, and IG process are considered in this study $$Y_i(t) = \mu_i t + \sigma_i B_i(t)$$ $$f_{\mathcal{IG}}(y;a,b) = \left(\frac{b}{2\pi y^3}\right)^{\frac{1}{2}} \exp\left\{-\frac{b(y-a)^2}{2a^2 y}\right\}, y>0$$ $$f_{\mathcal{Ga}}(y;a,b) = \frac{b^a}{\Gamma(a)} y^{a-1} e^{-by}, y>0$$
Kıvanç et al (2022) [25]	$V*(b)$ is the optimal value function maximizing the expected total revenue of the process on the belief space in the long run for belief b which is determined by dynamic programming $$V^*(b) = \max_{a\in\mathcal{A}} \left\{ \sum_{s\in S} \rho(s,a)b(s) + \gamma \sum_{o\in\mathcal{O}} \sum_{s\in S} P(o\|s,a)b(s)V^*(b') \right\}$$ $$\rho(s,a) = \sum_{s'\in S} \sum_{o\in\mathcal{O}} R(s,a,s',o)P(s'\|s,a)P(o\|s',a)$$
Tambe (2022) [26]	The total cost incurred due to a maintenance decision is the sum of the maintenance actions and the total failure cost, which is given as $$\text{Total Maintenance Decision Cost (TMDC)} = \sum \text{Repair Cost} + \text{Replacement Cost} + \text{Failure Cost}$$ Repair cost $$\mathrm{TC}_{\boldsymbol{r}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} [\boldsymbol{Y_i} \times (\boldsymbol{C_r})_{\boldsymbol{i}}]$$ $$(C_r)_i = [\mathrm{MTTrA}_{\ i} \times (\mathrm{PR} \times C_{\mathrm{LP}} + C_{\mathrm{Labor}}) + (C_s)_i\ ]$$ Replacement cost $$\mathrm{TC}_{\boldsymbol{Replace}} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} [\boldsymbol{X_i} \times (\boldsymbol{C_R})_{\boldsymbol{i}}]$$ $$(C_R)_i = [\mathrm{MTTRA}_{\ i} \times (\mathrm{PR} \times C_{\mathrm{LP}} + C_{\mathrm{Labor}}) + \mathrm{C}_{\ i} + \mathrm{CRL}_i]$$
Wang et al (2022) [27]	Degradation level of the unmaintained component j modeled by a basic Wiener process $$X_j(t) = \mu_j t + \sigma_j \mathcal{B}(t)$$ The RUL of the jth component is governed by an IG distribution with the PDF as $$f_{r_j(u\|\tilde{x}_j)}(v) = \frac{L_j - \tilde{x}_j}{\sqrt{2\pi\tilde{\sigma}_j^2 v^3}} \exp\left(-\frac{(L_j - \tilde{x}_j - \mu_j v)^2}{2\tilde{\sigma}_j^2 v}\right).$$
Oakley et al (2022) [28]	The degradation of Component ij at t_k $$D_{ij}(t_k) = \int_{t_{ij}^*}^{t_k} \Delta D_{ij}(t;\theta_{D_{ij}}, x_i(t), \Pi_i(t) = \pi_i(t))dt$$ t_{ij}^* is the time of the most recent replacement of Component ij

Zhou et al (2016) [29]	The conditional transition probability of the system is simplified as: $$\Pr(\mathbf{X}'_s \mid \mathbf{X}_s, \mathbf{A}_s) = \prod_{n=1}^{N_s} \Pr(X'_{un}, X'_{dn} \mid X_{un}, X_{dn}, A_{un}, A_{dn})$$ $$\Pr(X'_{un}, X'_{dn} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = \Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) \cdot \Pr(X'_{dn} \mid X_{un}, X_{dn}, A_{un}, A_{dn})$$ Situation one: $$\Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = (\mathbf{P}_{\mathbf{un},X_{dn}})_{X_{un}X'_{un}}$$ Situation two: $$\Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = I(X'_{un} = X_{un})$$ Situation three: $$\Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = I(X'_{un}=1)P_{p,un} + I(X'_{un}=S_{un}-1)(1 - P_{p,un})$$ Situation four: $$\Pr(X'_{un} \mid X_{un}, X_{dn}, A_{un}, A_{dn}) = I(X'_{un}=1)P_{c,un} + I(X'_{un}=S_{un})(1 - P_{c,un})$$

Developing a new multi-component model

In this section, we assume that a system constitutes n identical components that component 1 is connected to component 2 and component 2 is connected to component 3, etc. Now, let consider n=3 as in Figure 1.

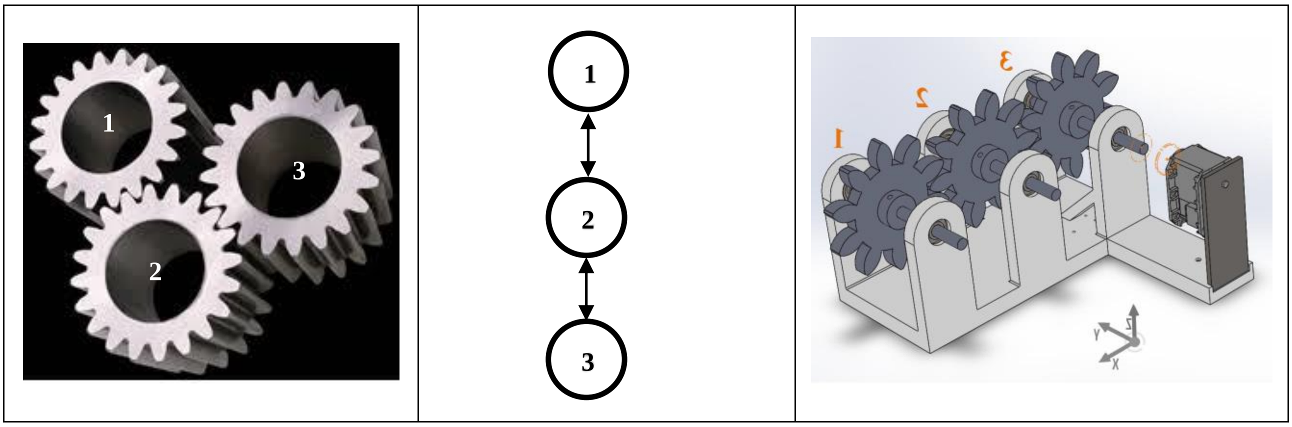


Fig 1. The assumed multi-component system, n=3

Now, let's consider the degradation function as in following:

$$x_k^{(i)} = x_{k-1}^{(i)} + \eta^{(i)}\Delta t_{k-1} + \alpha_1 x_k^{(i-1)} + \alpha_2 x_k^{(i+1)} + \alpha_3 x_k^{(i-1)} x_k^{(i+1)} + w_{k-1}^{(i)}$$

$i = 1, \dots, n$ (2)

In which:

$w_{k-1}^{(i)} = \sigma_B^{2(i)} B(t)$, $\Delta t_{k-1} = 1$

$\eta^{(i)}$= the Constant inherent degradation rate of each component

$x_k^{(i)}$= the degradation value of component i in at t_k

$\alpha_1, \alpha_2, \alpha_3$ = the Constant coefficients

$x_k^{(i-1)} x_k^{(i+1)}$ = the mutual effect of destroying the components $(i-1)$ and $(i+1)$ on component i

$w_{k-1}^{(i)}$ = the state transition noise of component i, $w_{k-1}^{(i)} \sim N\left(0, \sigma_B^{2(i)} \Delta t_k\right)$

$\sigma_B^{2(i)}$ = diffusion coefficients of component i

$B(t)$= standard Brownian motion

Numerical studies

In this section, a simulation case study is conducted to demonstrate our developed model. The proposed system has three components and each component has a sensor to monitor the degradation trajectory. So, we simulated three degradation datasets. Each data set constitutes 100 degradation trajectories and in general, has 20631 data for each component. The degradation trajectories for the component are shown in Figure 2.

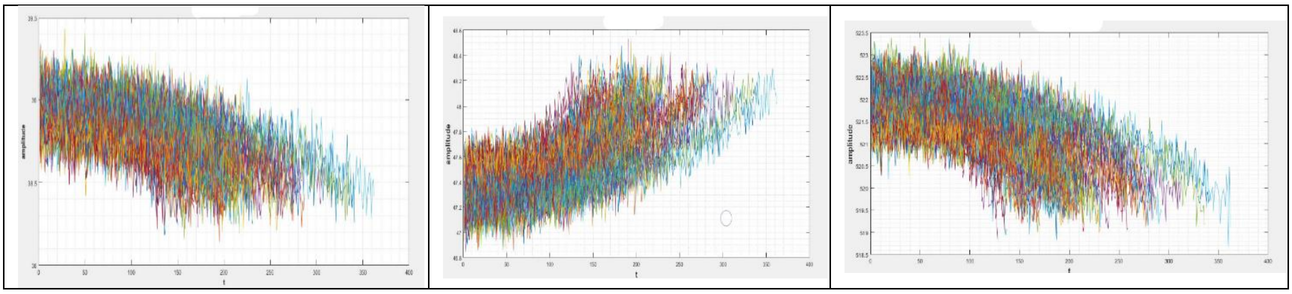


Fig 2. Degradation trajectories for components 1, 2 and 3 (left to right) for 100 units (smoothed curves presented)

Now we estimate the parameters of the model using the least square method. To estimate the parameters, we may rewrite the equation too as:

$$\rightarrow \quad \Delta x_k{}^{(1)} = x_k^{(i)} - x_{k-1}^{(i)} = \eta^{(i)}\Delta t_{k-1} + \alpha_1 x_k^{(i-1)} + \alpha_2 x_k^{(i+1)} + \alpha_3 x_k^{(i-1)} x_k^{(i+1)} + w_{k-1}^{(i)} \tag{3}$$

The obtained models for each component are as follows:

For component 1:

$$\Delta x_k{}^{(1)} = \eta + \alpha x_k^{(2)} + w_{k-1}^{(1)} \quad \rightarrow \quad \Delta x_k{}^{(1)} = 1.6827 - 0.035 x_k^{(2)} + w_{k-1}^{(1)}$$

$$w_{k-1}^{(1)} \sim N\left(0, \sigma_B^{2\,(1)} \Delta t_k\right) \quad , \quad w_{k-1}^{(1)} \sim N(0, 0.0202)$$

For component 2:

$$\Delta x_k{}^{(2)} = \eta + \alpha_1 x_k^{(1)} + \alpha_2 x_k^{(3)} + \alpha_3 x_k^{(1)} x_k^{(3)} + w_{k-1}^{(2)} \rightarrow$$

$$\Delta x_k{}^{(2)} = 91.7988 - 1.8533 x_k^{(1)} - 0.1887 x_k^{(3)} + 0.0039 x_k^{(1)} x_k^{(3)} + w_{k-1}^{(2)}$$

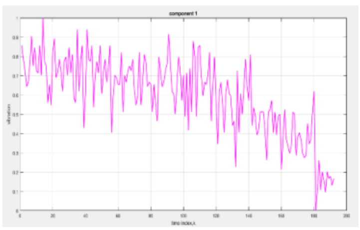
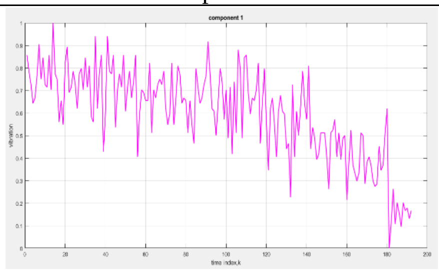
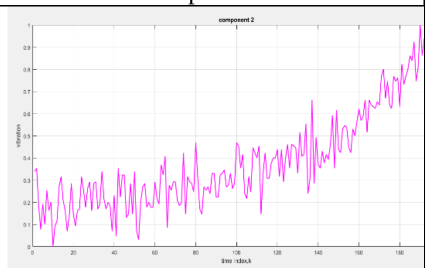
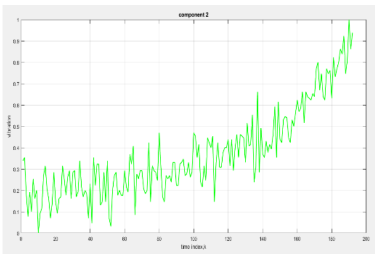
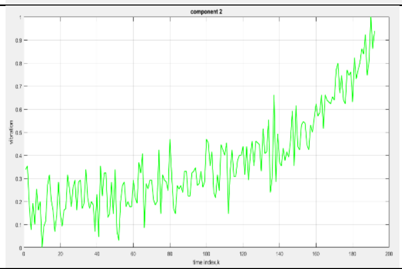
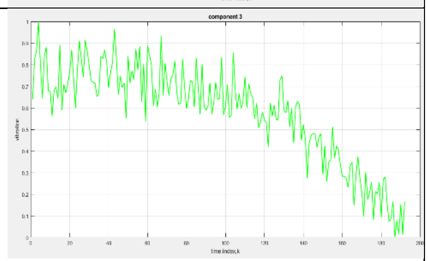
$$w_{k-1}^{(2)} \sim N\left(0, \sigma_B^{2\,(2)} \Delta t_k\right) \quad , \quad w_{k-1}^{(2)} \sim N(0, 0.024)$$

For component 3:

$$\Delta x_k{}^{(3)} = \eta + \alpha x_k^{(2)} + w_{k-1}^{(3)} \quad \rightarrow \quad \Delta x_k{}^{(3)} = -0.4892 + 0.0101 x_k^{(2)} + w_{k-1}^{(3)}$$

$$w_{k-1}^{(3)} \sim N\left(0, \sigma_B^{2\,(3)} \Delta t_k\right) \quad , \quad w_{k-1}^{(3)} \sim N(0, 0.1736)$$

According to the obtained models, the estimated RUL plot for each component is shown in Figure 3.

Component 1	Component 2	Component 3
		
		

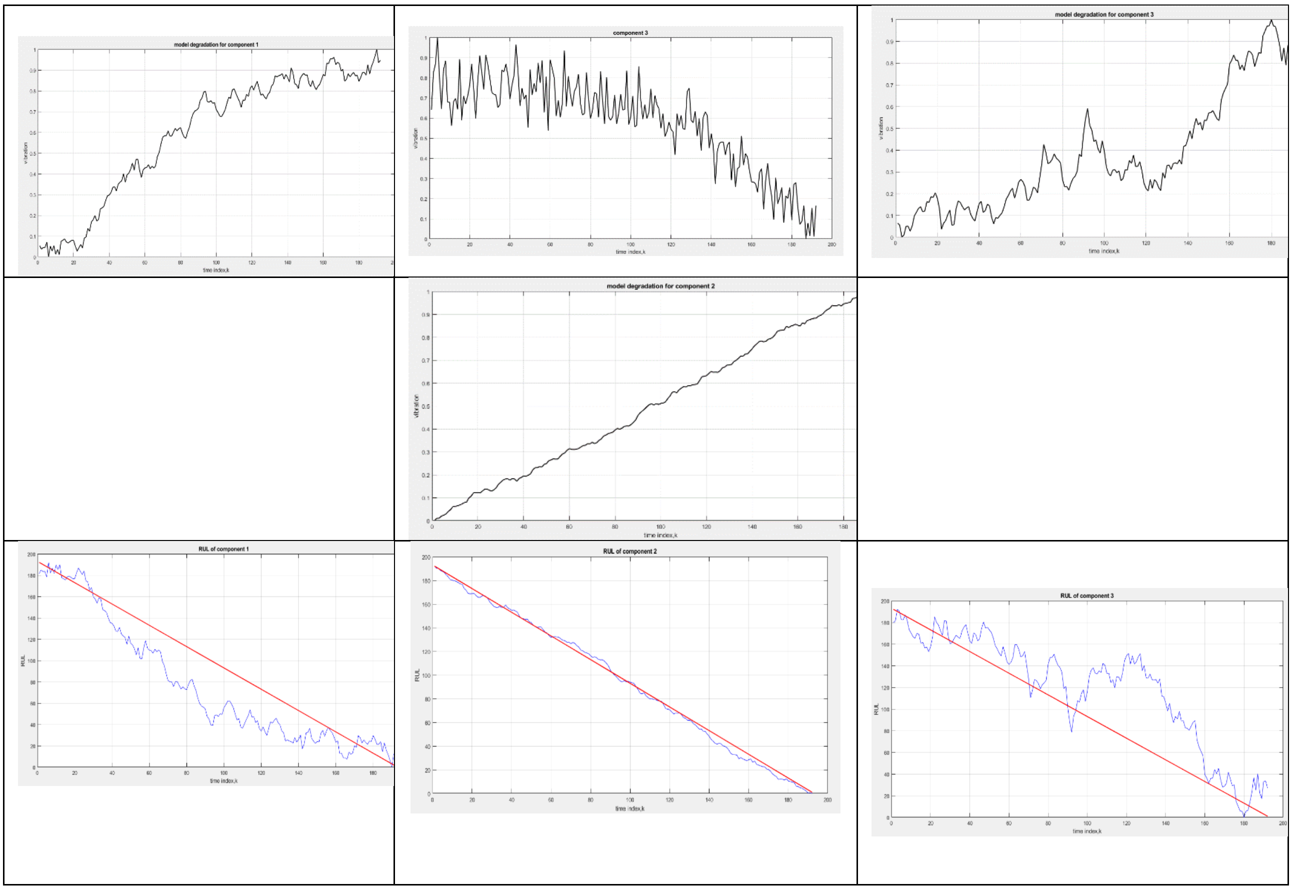


Fig 3. RUL for each component

As is shown in Figure 1, the studied system is a series system with respect to reliability theory. The series system fails when at least one of its components fails. Thus, the predicted RUL of this system is obtained by finding the minimum among the RUL predictions of the three components. The results are shown in Figure 4

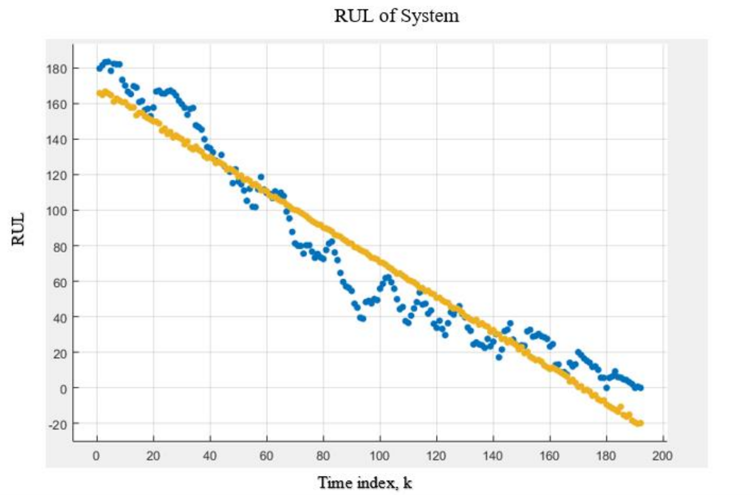


Fig 4. RUL of the system

Conclusion

In this paper, we presented a degradation model for a multi-component system. A Brownian motion was considered to describe the effect of environmental factors. To estimate the unknown parameters, we used the least square method. After obtaining the RUL of each component, the RUL of the whole system was obtained.

Reference

1. Dekker R, Wldeman RE, Schouten D (1997). "A review of multi-component maintenance models with economic dependence". Mathematical Methods of Operations Research. 45:411-435
2. Nicolai R, Dekker R (2008). "Optimal Maintenance of Multi-component Systems: A Review". Complex System Maintenance Handbook. pp 263–286
3. Cao W, Jia X, Hu Q, Zhao J, Wu Y (2018)." A literature review on selective maintenance for multi-unit systems". RESEARCH ARTICLE. 1–22
4. Zhang H, Chen M, Zhou D (2016). "Predicting Remaining Useful Life for a Multi-Component System with Public Noise". Prognostics and System Health Management Conference (PHM-Chengdu)
5. Bian L, Gebraeel N (2014). "Stochastic Modeling and Real-Time Prognostics for Multi-Component Systems with Degradation-Rate-Interactions". IIE Transactions.
6. Sheu SH, Liu TH, Zhang ZG, Chien YH (2015). "Extended Optimal Replacement Policy for a Two-Unit System with Shock Damage Interaction". IEEE TRANSACTIONS ON RELIABILITY
7. Zhang N, Fouladirad M, Barros A (2018). "Optimal imperfect maintenance cost analysis of a two-component system with failure interactions". Reliability Engineering and System Safety. doi: 10.1016/j.ress.2018.04.019
8. Dinh DH, Do P, Iung B (2020). "Degradation modeling and reliability assessment for a multi-component system with structural dependence". Computers & Industrial Engineering
9. Bakir I, Yildirim M, Ursavas E (2021). "An integrated optimization framework for multi-component predictive analytics in wind farm operations & maintenance". Renewable and Sustainable Energy Reviews.
10. Niu H, Zeng J, Shi H, Zhang X, Liang J (2022). "Degradation Modeling and Remaining Useful Life Prediction for a Multicomponent System with Stochastic Dependence". Computers & Industrial Engineering
11. Özgür-Ünlüakın D, Turkali B (2021). "Evaluation of proactive maintenance policies on a stochastically dependent hidden multi-component system using DBNs". Reliability Engineering and System Safety
12. Zhang N, Cai K, Zhang J, Wang T (2022). "A condition-based maintenance policy considering failure dependence and imperfect inspection for a two-component system". Reliability Engineering and System Safety.
13. Nguyen KTP, Medjaher K, Gogu C (2022). "Probabilistic deep learning methodology for uncertainty quantification of remaining useful lifetime of multi-component systems". Reliability Engineering and System Safety
14. Li H, Zhu W, Dieulle L, Deloux E (2022). "Condition-based maintenance strategies for stochastically dependent systems using Nested Lévy copulas". Reliability Engineering and System Safety
15. Shi H, Zeng J (2016). "Real-time prediction of remaining useful life and preventive opportunistic maintenance strategy for multi-component systems considering stochastic dependence". Computers & Industrial Engineering
16. Zhang N, Si W (2020)" Deep Reinforcement Learning for Condition-Based Maintenance Planning of Multi-Component Systems Under Dependent Competing Risks". Reliability Engineering and System Safety
17. Shen J, Elwany A, Cui L (2018). "Reliability analysis for multi-component systems with degradation interaction and categorized shocks" Applied Mathematical Modelling
18. Martinod RM, Bistorin O, Castaneda LF, Rezg N (2018). "Maintenance policy optimisation for multi-component systems considering degradation of components and imperfect maintenance actions". Computers & Industrial Engineering
19. Xu M, Jin X, Kamarthi S, Noor-E-Alam Md (2018). "A failure-dependency modeling and state discretization approach for condition-based maintenance optimization of multi-component systems". Journal of Manufacturing Systems
20. Nguyen KA, Do P, Grall A (2017). "Joint predictive maintenance and inventory strategy for multi-component systems using Birnbaum's structural importance". Reliability Engineering and System Safety
21. Do P, Assef R, Scarf P, Lung B (2018) "Condition-based maintenance for a two-component system with stochastic and economic dependencies". Reliability Engineering and System Safety
22. Shafiee M, Labib A, Maiti J, Starr A (2019). "Maintenance strategy selection for multi-component systems using a combined analytic network process and cost-risk criticality model". Institution MECHANICAL ENGINEERS
23. Liu B, Pandey MD, Wang X, Zhao X (2021). "A finite-horizon condition-based maintenance policy for a two-unit system with dependent degradation processes". European Journal of Operational Research
24. Xu J, Liang Z, Li YF, Wang K (2021). "Generalized condition-based maintenance optimization for multi-component systems considering stochastic dependency and imperfect maintenance". Reliability Engineering and System Safet
25. Kıvanç I, Özgür-Ünlüakın D, Bilgiç T (2022). "Maintenance policy analysis of the regenerative air heater system using factored POMDPs". Reliability Engineering and System Safety
26. Tambe PP (2022). "Selective Maintenance Optimization of a Multi-component System based on Simulated Annealing Algorithm". Procedia computer Science
27. Wang Y, Li X, Chen J, Liu Y (2022). "A condition-based maintenance policy for multi-component systems subject to stochastic and economic dependencies". Reliability Engineering and System Safety.
28. Oakley JL, Wilson KJ, Philipson P (2022). "A condition-based maintenance policy for continuously monitored multi-component systems with economic and stochastic dependence". Reliability Engineering and System Safety
29. Zhou Y, Lin TR, Sun Y, Ma L (2016). "Maintenance optimisation of a parallel-series system with stochastic and economic dependence under limited maintenance capacity". Reliability Engineering and System Safet