Combined Markov and UGF Method for Multi-State Repairable Phased Mission Systems

Fatma Beria OKTAY[1], Özge ELMASTAŞ GÜLTEKİN [2*]

1-Department of Statistics, Ege University, İzmir, Turkey
2-Department of Statistics, Ege University, İzmir, Turkey

[*] ozge.elmastas@ege.edu.tr

Abstract
The reliability analysis of multi-state phased mission systems (MS-PMS) is a crucial area of study in systems engineering and reliability engineering. A MS-PMS consists of multiple phases where the system can exist in different operational states in each phase. The system transitions from one phase to the next based on the success or failure of the current phase. The reliability of a MS-PMS depends on the reliabilities of each phase as well as the transition probabilities between system states across phases. By thoroughly analyzing the reliability of each phase and accurately estimating the probabilities of state transitions, it can be determined the overall system reliability. There are several methods used for MS-PMS reliability analysis such as Markov models, Universal Generating Function (UGF) technique, Petri nets, fault trees, etc. In this study, reliability analysis of a MS-PMS was evaluated with a combination of Markov and UGF techniques. This method is defined as a combined technique in the literature. The Markov modelling approach represents the system as a set of states with transitions between states based on the failure and repair of components. In addition, the UGF technique converts the Markov model into a set of algebraic equations that can be solved to obtain reliability metrics such as system availability, mean time to failure, etc. In this research, a three phased multi-state repariable system was discussed. For all phases, transition diagrams were created on the basis of components, and the resulting differential equations were solved. Then, the UGF method was applied according to the system structure of the phases and the reliability metrics of the system was obtained.

Keywords: Markov, Multi-state systems, Phased mission, Repairable, UGF

Introduction

Phased Mission System (PMS) is a reliability concept that has gained significant attention in recent years. It is based on the idea of dividing a mission into different phases, each with its own requirements and objectives. The use of PMS is becoming increasingly popular in various industries, including aviation, space, defense, and automotive. It is believed to be an effective tool for enhancing system reliability by identifying potential issues early on and addressing them before they become significant problems. The concept of PMS was first introduced by NASA in the 1970s for space missions. Since then, it has been widely adopted and used in various industries. Several studies have been conducted to evaluate the effectiveness of PMS in increasing system reliability. There are several methods used in the literature to obtain reliability analysis of phased mission systems. [1] proposed a methodology for analyzing the reliability of phased mission systems. It's approach is based on the assumption that the system can be divided into independent phases, each with its own reliability parameters. The methodology proposed in [1] provides a useful framework for evaluating the reliability of complex systems that pass through multiple working phases. The model allows for the identification of critical phases to improve the overall system reliability and optimize maintenance and repair activities. [2] conducted a study on the reliability analysis of phased mission systems using Binary Decision Diagrams (BDDs) and aimed to develop a new model that considers the different phases and dependencies of a system during a mission. The proposed model uses BDDs to represent the possible states of the system and transitions between them, which provides a more accurate estimation of the system's reliability. [3] proposed a new method for the reliability analysis of non-repairable phased mission systems using Multiple-valued Decision Diagrams (MDDs) and aimed to evaluate the system's reliability more efficiently and accurately by considering the dependencies between system components and different phases of the mission.

[4] has proposed a new reliability model that takes into account the dependencies between different phases of a mission and system components, allowing for the estimation of the system's reliability and error scope at each phase of the mission. Additionally, it can be used to identify the critical components of the system. [5] has proposed a non-homogeneous Markov model for reliability analysis of phased mission systems. The model aims to more accurately estimate the reliability of a system by considering the changes in stress levels and failure rates of the system during different phases of a mission. The proposed model has been shown to effectively estimate a system's reliability at each phase of the mission and can be used to identify the critical components of the system. However, considering the non-homogeneous nature of the system, it allows for a more comprehensive evaluation of the system's reliability.

In Phased Mission Systems, each component can exhibit multiple performance levels or states not only between phases but also within the same phase. These systems are called Multi-State Phased Mission Systems (MS-PMS). Reliability analysis methods for MS-PMS are used to evaluate the reliability of systems that have multiple possible states and transition between phases during a mission. These methods take into account the possible transitions between different phases and states of the system to estimate its reliability. Various models and methods have been proposed for the reliability analysis of MS-PMS, including Binary Decision Diagrams (BDDs), Multi-Valued Decision Diagrams (MDDs), Markov models and other methods [6-9].

In addition to the methods used in the calculation of the reliability of multi-state systems in the literature, Markov and UGF methods are considered in combination. The combined method was first introduced in [10]. The universal generating function allows for the simple algebraic manipulation of the production functions of a multi-state system with components connected in series or parallel, by obtaining the individual production function of the multi-state component [11]. In this study, the combined Markov and UGF technique was applied to obtain reliability measures for repairable multi-state phased mission systems. The application of the combined method is demonstrated for a three-component, three-phase, and three-state repairable system.

Repairable Systems

Systems can be classified as repairable and non-repairable systems. The reliability analysis of repairable systems is an important research area for the design and development of critical systems used in engineering, aviation, defense, and transportation. A repairable system is a system in which, after failing to perform at least one of its intended missions, it is possible to continue the system's operation by repairing the component that caused the failure instead of replacing the entire system [12]. The reliability of these systems depends on various factors such as the frequency and duration of repair events, the effectiveness of the repair process, and the quality of the repair materials used.

Different models and methods have been proposed in the literature for reliability analysis of repairable systems, including Renewal Theory, Markov Model, and Weibull Distribution. These methods are used to estimate the system's reliability, identify critical components, optimize the system's structure to increase its reliability and ensure its efficient operation. Renewal Theory is a probabilistic model used to analyze the behavior of a repairable system, especially the frequency and duration of repair events. The model includes estimating the failure rate and repair rate of the system, and finding the probability distribution of the time between failures. Markov model is another method used in reliability analysis of repairable systems, which involves representing the system as states and transitions and analyzing the probability of transitioning between these states. Weibull distribution is another method used in reliability analysis of repairable systems, which is used to estimate the probability distribution of time between failures and identify the critical components of the system. Reliability analysis of repairable systems is a way to ensure the safe and efficient operation of systems critical to mission success. By using these methods and models, repairable systems can be designed, developed, and sustained to the highest possible standards, reducing the risk of failure and improving system performance [13-15].

Multi-state Systems

A set of related components designed for a specific purpose, working together or within a single unit, is defined as a system. Reliability, on the other hand, is the probability of a product/system to perform its intended function adequately during the intended period under the encountered operating conditions. Reliability studies are divided into traditional binary reliability models, which allow for only two possible scenarios for a system and its components: perfect performance and complete failure. However, the increasing dependency on devices in today's changing and evolving world has led to a complex system situation in [16]. Many real-world systems have been designed to perform their missions with various distinctive levels of efficiency called performance ratios. Systems with a finite number of performance ratios are called multi-state systems [17]. Multi-state systems consist of components that have different performance ratios, one after the other. In reliability systems, the operation of the system depends on one or more components, and the reliability of these components determines the reliability of the system. The most commonly used methods in the reliability calculations of multi-state systems in the literature are Markov and UGF methods.

Phased Mission Systems (PMS)

With the increasing demands and technological advancements, system structures have become more complex. Applications in fields such as aviation, nuclear energy, electronics, and transportation often involve several different missions or phases that need to be carried out sequentially. These systems are referred to as phased mission systems [18]. Analyzing phased mission systems is complex compared to single-phase systems due to the system structure changing between phases and the different component failures in different phases being interdependent. The mission process carried out by these systems can be divided into several consecutive phases that include different subsystems or components. For example, an aircraft system needs to go through takeoff, climb, level flight, descent, and landing phases [19]. During each mission phase, the system must perform a specific mission and has different reliability requirements. To analyze the reliability of phased mission systems, it is necessary to first examine and understand previous periods of the system. Afterward, reliability metric variables are defined, and these metric variables are calculated using one of the existing reliability calculation methods. The reliability of phased mission systems can be generally defined as the probability of successfully achieving the targeted mission in all phases [20]. Figure 1 below shows a practical and simple example of a phased system structure consisting of three phases and three components.

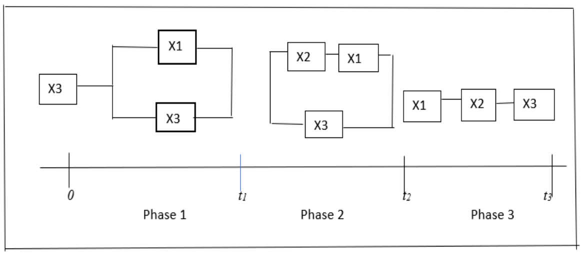

Fig.1. Example of a Three Phase and Three Component Phased Mission System

Methods Used in Reliability Calculations

Most commonly used methods for system reliability analysis, such as fault tree analysis, binary decision diagram, Markov method, UGF method, and combined Markov and UGF method, etc.

Fault Tree Analysis

The fault tree analysis is usually used in the design phase of a system to emphasize the improvement areas in the reliability of the system and its use by operators. The fault tree analysis was first developed at Bell Telephone Laboratories in 1962 to facilitate the analysis of the intercontinental Minuteman missile launch control system. Later, it was developed and implemented by Boeing. Nowadays, fault tree analysis is used as a logical analytical technique in system reliability studies, especially in nuclear power plants, aviation, and defense systems.

In the fault tree analysis, first, an unwanted event is defined. For system reliability analysis, the unwanted event is generally the failure of the system or subsystem. Then the system is analyzed in terms of its environment and operation to find all basic event combinations that could lead to the occurrence of previously defined unwanted events. The basic event mentioned here are the events that could cause an unwanted event or failure, such as component failures, human or environmental failures. The logical relationship between the basic event and the unwanted event is graphically represented using the fault tree analysis, and a logical inference is made to understand how a system can fail [18].

Binary Decision Diagram

When applied to complex or large systems, the analysis methods used for decision trees can become inefficient. Instead of analyzing the system in this way, converting fault trees to binary decision diagrams and then analyzing them is more effective and efficient. Binary decision diagrams were originally used as compact encodings in circuit design and verification, instead of logical expressions known as Boolean expressions. It is difficult to obtain results from binary decision diagrams in their original form, so they are usually compared to fault trees to obtain results. Binary decision diagrams and fault tree models can be solved without requiring excessive computational complexity or time. Therefore, their use is effective in large decision tree models. A binary decision diagram consists of nodes where an event starts or ends, connected by branches. Events start from the top of the diagram and end in a node representing the occurrence or non-occurrence of an event, represented by 1 or 0 nodes respectively [21].

Markov Method

A system's development is represented by a continuous-time discrete-state stochastic process [22]. Stochastic process theory provides an advanced probabilistic framework that allows for the formulation of general failure models for real systems, obtaining explicit formulas for various reliability indices for calculations, and determining optimal maintenance plans in complex situations [11]. The Markov method is used when a system's components have strong dependencies. Markov modeling is a mathematical model that is known as the "Markov approach," named after mathematician Andrei Markov (1856-1922). It analyzes the future behavior of any system based on its past behavior, depending solely on the previous state's behavior, and is used in situations where a system's components have strong dependencies. This modeling can be done due to the "memoryless" feature of the Markov approach, and the above-mentioned approach assumes that the transition time from one state to another is constant, making the method suitable for reliability calculations. In a Markov model, there are two sections: states and transitions. While defining the states as the working or malfunctioning of the components in the system, transitions represent the transition time from one state to another. There are no transitions between some states, and thus, the transitions are not connected to each other. These states are called absorbing states [23]. It is assumed that the failure and repair times of the components are associated with exponential distributions due to stochastic process theory [24]. In Markov modeling, state-space matrices are first created for the system, and transitions between all states are defined. Then, the Markov state differential equations obtained are solved by methods such as Laplace transformation to find state probabilities. The disadvantage of this method is the high-dimensional problem, and it requires quite large computational resources. Therefore, it is more useful for smaller systems.

A popular type of Markov process used in reliability analysis is the Continuous Time Markov Chain (CTMC). CTMC is used to model systems that can transition between multiple states over time with transition rates specified by a set of parameters. It has been used to analyze the reliability of a wide range of systems, including communication networks, production systems, and power systems. Another type of Markov process commonly used in reliability analysis is the semi-Markov process. Semi-Markov processes are similar to CTMCs but allow for transitions to occur at non-exponential times. They are particularly useful for modeling systems with complex repair or maintenance programs where the duration between transitions may depend on the current state of the system. In addition to CTMCs and semi-Markov processes, other types of Markov process such as hidden Markov models and Bayesian networks have also been used in reliability analysis. Hidden Markov models are used to model systems where the true state of the system cannot be directly observed, while Bayesian networks can be used to model dependencies between components of a system [17].

The Markov transition diagram shown schematically in Figure 2 depicts a single repairable component with working and failed states.

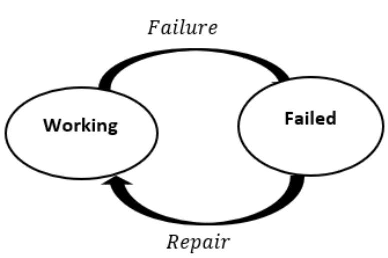

Fig.2. Markov Transition Diyagram for One Component

Figure 3 below shows a simple representation of the transition from any state s to state m for a multi-state repairable component using a Markov process. For $s, m = 1,2, \dots, k$; the failure rate is defined λ_{sm} as when $m<s$ and the repair rate is defined as μ_{sm} when $m>s$ for this multi-state component.

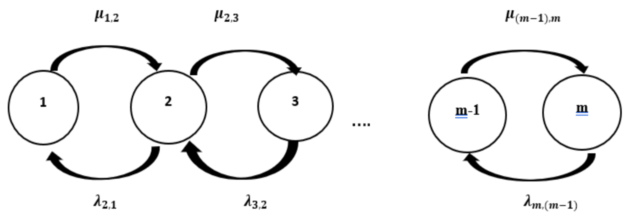

Fig.3. Markov Transition Diagram for a Multi-state Component

Universal Generating Function (UGF) Technique

UGF is a powerful mathematical tool used to analyze the probability distribution of the total number of failures that occur in a system within a certain period of time, taking into account the failure rates of individual components within the system. UGF was first introduced by Ushakov (1986, 1987) in studies of system reliability. This method is highly effective for high-dimensional combinatorial problems [11]. The universal generating function technique allows for the determination of the performance distributions of a system's components using algebraic procedures, allowing for the determination of the performance distribution of the entire system based on these distributions. One of the most significant benefits of the UGF approach is that it allows for a more quantitative and rigorous analysis of system reliability, providing valuable insights into the behavior of complex systems over time. The Equation 1 below shows the instantaneous performance distribution.

$$U(z,t) = \sum_{i=1}^{K} p_i(t)\, z^{g_i} \qquad (1)$$

A multi-state component i can have states representing different levels of performance. In this case, the performance set for this component is represented by Equation 2.

$$g_i = \{g_{i0}, g_{i1}, g_{i2}, \dots, g_{im}\} \qquad (2)$$

The availability of a system for any multi-state system with $t > 0$ and a given demand w is calculated using the following Equation 3.

$$A(t,w) = \delta_A(U(z,t), w) = \sum_{i=1}^{K} P_i(t)\delta(g_j \geq w) \qquad (3)$$

In the study conducted in [25], space systems were modeled analytically by dividing them into phases. In [26], the propulsion system of an ion propulsion system sent to the outer solar system for a scientific mission is a phased mission system consisting of seven phases. In the study, the time-dependent reliability of the mentioned propulsion system was aimed to be determined during the planned mission period. [27] evaluated the reliability of the attitude and orbit control system (AOCS), which is responsible for keeping the spacecraft in the correct position and orbit throughout its lifetime, using a phased mission system and Markov renewal equation-based method. [28] evaluated the reliability of a critical system used to adjust the direction and orbit in a spacecraft, the propulsion system, in their study. In [29], the risk elements of the Mars Smart Lander project (MSL-09) were effectively calculated with the fault decision tree method approach based on data obtained from experts. [30] introduced the power generation system as a phased mission system and showed that the existing epidemiological uncertainty due to the lack or inaccuracy of data increases over time and the system usability decreases when transmission loss is taken into account based on the study conducted on twenty-four bus power generation systems.

Multi-state Phased Missions System (MS-PMS)

MS-PMS is an innovative concept that has emerged in the field of reliability analysis. It is an extension of the Phased Mission System (PMS), which is based on the idea of dividing a mission into different phases, each with its own requirements and objectives. MS-PMS takes PMS one step further by allowing for multiple states within a phase, which can be particularly useful for complex systems or missions involving multiple phases. The use of MS-PMS can enable a more comprehensive understanding of system performance by allowing for the analysis of each state within a phase. There are various methods used for reliability analysis of multi-state phased mission systems, including Markov models, UGF method, combined Markov and UGF method, Petri nets, fault trees, Bayesian networks, and simulation models. In this study, the combined Markov and UGF method will be used for the example of the multi-state phased mission system addressed.

Combined Markov and UGF Method

Combined Markov and UGF method is a proposed approach for analyzing the reliability of Multi-State Phased Mission Systems (MS-PMS). This method combines the use of Markov models with UGF to provide a more comprehensive analysis of system performance. Markov models are commonly used to analyze the reliability of systems with discrete states. They allow for the estimation of the probability of a system being in a certain state at a specific time. UGF, on the other hand, is used to analyze the reliability of systems with continuous states.

The Combined Markov and UGF method allows for the analysis of MS-PMS systems with both discrete and continuous states. This approach can provide a more accurate understanding of system performance by considering all possible states and transitions. The following are the steps involved in the Combined Method:

1. Represent the system using a Markov model with states corresponding to the status of components.
2. Convert the Markov model into a set of differential equations based on transition rates between states.
3. Convert the differential equations into algebraic equations by applying the UGF technique.
4. Solve the algebraic equations to obtain the reliability metrics of interest.

Combined Markov and UGF Method for Repairable Three-state Three Component and Three Phased Mission Systems

In this study, we want to show how the combined Markov and UGF technique for multistate systems can be applied on a three-phase, three-component and three-state repairable phased mission system. The structure of the system and the application steps of the method are given below.

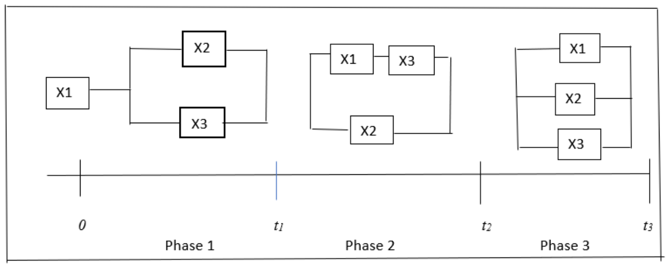

Fig.2. Phased mission system with three phases, three components and three states

Assumptions made for the phased mission system are listed below:

1. The system consists of a three-state, three-component, and three-phase mission system.
2. The transition time between two consecutive phases is not important.
3. All components work perfectly at the beginning.
4. Components are three-state (perfectly working (2)- working (1)- failed (0)) and repairable.
5. There is only one repairman in the system.
6. Repair can only be done when the system is working. After being repaired, each component is "as good as new".
7. The repair and failure rates of the components are independent random variables and are exponentially distributed.

Application of Combined Method

First, state transition matrices are created for each component in Markov modeling, and transitions between all possible states are defined. Then, differential equations are established to determine the probabilities of each component state. Figure 3 shows the transition diagram for a 3-state component, and Table 1 provides the parameter table for the components.

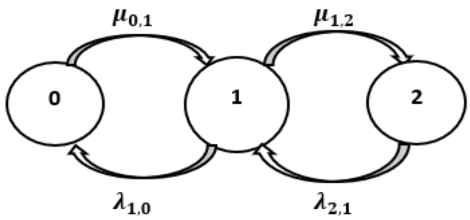

Fig.3. State Transition Diagram for a 3-state component

Table 1. The Parameters of Components in the System

Component(i)	States(j)	Performance (g_j^i)	Failure Rate ($\lambda^i_{j,j-1}$)	Repair Rate ($\mu^i_{j,j+1}$)
X_1	0	0	----	0.4
	1	1.5	0.07	0.4
	2	2	0.07	----
X_2	0	0	----	0.4
	1	2	0.07	0.4
	2	2.5	0.07	----
X_3	0	0	----	0.4
	1	1.8	0.07	0.4
	2	4	0.07	----

Differantial equations for component X_1,

$$\begin{cases} \dfrac{dp_0^{X_1}(t)}{dt} = -\mu_{0,1}P_0^{X_1}(t) + \lambda_{1,0}P_1^{X_1}(t) \\ \dfrac{dp_1^{X_1}(t)}{dt} = \mu_{0,1}P_0^{X_1}(t) - (\mu_{1,2} + \lambda_{1,0})P_1^{X_1}(t) + \lambda_{2,1}P_2^{X_1}(t) \\ \dfrac{dp_2^{X_1}(t)}{dt} = \mu_{1,2}P_1^{X_1}(t) - \lambda_{2,1}P_2^{X_1}(t) \end{cases}$$

(4)

for component X_2,

$$\begin{cases} \dfrac{dp_0^{X_2}(t)}{dt} = -\mu_{0,1}P_0^{X_2}(t) + \lambda_{1,0}P_1^{X_2}(t) \\ \dfrac{dp_1^{X_2}(t)}{dt} = \mu_{0,1}P_0^{X_2}(t) - (\mu_{1,2} + \lambda_{1,0})P_1^{X_2}(t) + \lambda_{2,1}P_2^{X_2}(t) \\ \dfrac{dp_2^{X_2}(t)}{dt} = \mu_{1,2}P_1^{X_2}(t) - \lambda_{2,1}P_2^{X_2}(t) \end{cases}$$

(5)

for component X_3,

$$\begin{cases} \dfrac{dp_0^{X_3}(t)}{dt} = -\mu_{0,1}P_0^{X_3}(t) + \lambda_{1,0}P_1^{X_3}(t) \\ \dfrac{dp_1^{X_3}(t)}{dt} = \mu_{0,1}P_0^{X_3}(t) - (\mu_{1,2} + \lambda_{1,0})P_1^{X_3}(t) + \lambda_{2,1}P_2^{X_3}(t) \\ \dfrac{dp_2^{X_3}(t)}{dt} = \mu_{1,2}P_1^{X_3}(t) - \lambda_{2,1}P_2^{X_3}(t) \end{cases}$$

(6)

The differential equations obtained from the equations (4), (5) and (6) were solved by the Runge-Kutta method in Matlab program by updating the initial parameters for each phase according to the probabilities of the components in the previous phase. For Phase 1, the initial condition of each component is taken as $P_0^{X_i}(0)=0$, $P_1^{X_i}(0)=0$, $P_2^{X_i}(0)=1$, $i=1,2,3$ and the time intervals for each phase are taken as equal (0, $t_1=2$, $t_2=4$, $t_3=6$).

In Equation 7 below, UGF tranformation is shown for the relevant example according to the relevant performance values on a component basis.

$$u_i(z) = \sum_0^j p_i\, z^{g_j^i} \qquad i = 1,2,3\,; j = 0,1,2 \qquad (7)$$

For Phase 1:

Let's consider the completion of Phase 1 phase of the system at the time between t_1=0-2. According to the performance output of the system structure, the demand constant that Phase 1 must meet is determined as $\boldsymbol{w_1=1.5}$. The u transformations for each component are shown in Equation 8.

$$\begin{cases} u_1(z) = p_0^{X_1}(t)z^{g_0^{X_1}} + p_1^{X_1}(t)z^{g_0^{X_1}} + p_2^{X_1}(t)z^{g_0^{X_1}} \\ u_2(z) = p_0^{X_2}(t)z^{g_0^{X_2}} + p_1^{X_2}(t)z^{g_0^{X_2}} + p_2^{X_2}(t)z^{g_0^{X_2}} \\ u_3(z) = p_0^{X_3}(t)z^{g_0^{X_3}} + p_1^{X_3}(t)z^{g_0^{X_3}} + p_2^{X_3}(t)z^{g_0^{X_3}} \end{cases} \qquad (8)$$

In the system structure in Phase 1, X_2 and X_3 components are in parallel (P) structure and X_1 component is connected to them in series (S). In this case, for the system;

$$U_{sys} = (u_2(z) \otimes_{\emptyset P} u_3(z)) \otimes_{\emptyset S} u_1(z)$$

(9)

Phase 1 sistem availability for constant demand $w_1=1.5$ is

$$A(t, w_1) = \sum_{j=0}^{4} P_j(t)\delta(g_j \geq w_1 = 1.5) \cong 0.98$$

For Phase 2:

Let's consider the completion of Phase 2 phase of the system at t_2=2-4. According to the performance output of the system structure, the demand constant that Phase 2 must meet is determined as $\boldsymbol{w_2=3.5}$. The u transformations for each component are shown in Equation 10.

$$\begin{cases} u_1(z) = p_0^{X_1}(t)z^{g_0^{X_1}} + p_1^{X_1}(t)z^{g_0^{X_1}} + p_2^{X_1}(t)z^{g_0^{X_1}} \\ u_2(z) = p_0^{X_2}(t)z^{g_0^{X_2}} + p_1^{X_2}(t)z^{g_0^{X_2}} + p_2^{X_2}(t)z^{g_0^{X_2}} \\ u_3(z) = p_0^{X_3}(t)z^{g_0^{X_3}} + p_1^{X_3}(t)z^{g_0^{X_3}} + p_2^{X_3}(t)z^{g_0^{X_3}} \end{cases} \qquad (10)$$

In the system structure in Phase 2, X_1 and X_3 components are in series and X_2 component is connected in parallel to the serial structure. In this case, for the system;

$$_{sys} = (u_1(z) \otimes_{\emptyset S} u_3(z)) \otimes_{\emptyset P} u_2(z) \qquad (11)$$

Phase 2 sistem availability for constant demand w_2=*3.5* is

$$A(t,w_2)=\sum_{j=0}^{10}P_j(t)\delta\big(g_j\geq w_2=3.5\big)\cong 0.96$$

For Phase 3:

Let's consider the completion of Phase 3 phase of the system in time between t_3=4-6. According to the performance output of the system structure, the demand constant that Phase 3 must meet is determined as ***w*₃=7.5**. The *u* transformations for each component are shown in Equation 12.

$$\begin{cases} u_1(z)=p_0^{X_1}(t)z^{g_0^{X_1}}+p_1^{X_1}(t)z^{g_0^{X_1}}+p_2^{X_1}(t)z^{g_0^{X_1}} \\ u_2(z)=p_0^{X_2}(t)z^{g_0^{X_2}}+p_1^{X_2}(t)z^{g_0^{X_2}}+p_2^{X_2}(t)z^{g_0^{X_2}} \\ u_3(z)=p_0^{X_3}(t)z^{g_0^{X_3}}+p_1^{X_3}(t)z^{g_0^{X_3}}+p_2^{X_3}(t)z^{g_0^{X_3}} \end{cases} \quad (12)$$

In the system structure in Phase 3, X_1, X_2 and X_3 components are connected in parallel. In this case, for the system;

$$U_{sys}=(u_1(z)\otimes_{\emptyset P}u_3(z))\otimes_{\emptyset P}u_2(z)$$

(13)

Phase 3 sistem availability for constant demand w_3=*7.5* is

$$A(t,w_3)=\sum_{j=0}^{15}P_j(t)\delta\big(g_j\geq w_3=7.5\big)\cong 0.80$$

Results

There are numerous studies on the analysis of reliability metrics of multi-state systems, which are widely used in the field of engineering, using combined Markov and UGF method. However, no study has been found on the application of this combined method to repairable multi-state phased mission systems. This method provides ease of application in cases where the number of components and states is high.

In this study, for a three-phase, three-component, and three-state repairable phased mission system, when the probabilities of the components are considered in relation to the phase times, it is observed that the availability values, one of the metrics of system reliability, decrease as expected. In the continuation of this study, calculations will be made with different time intervals and different failure and repair rates for each phase, and comparisons will be interpreted.

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